Quiz #2 Resubmission

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We are asked to derive the moment-generating function of the exponential distribution. I had done exactly this during recitation on 10 September (correctly), and had derived a different moment-generating function (for the geometric) for exercise #70 in Chapter 2 of Ross's *Introduction to Probability Models* (part of homework #2), but apparently that wasn't enough practice to get it right "cold" under time pressure, because I did poorly on the quiz. I humbly resubmit a better answer now.

Suppose we have an exponentially distributed random variable X with the probability density function $f_X(x) = \lambda \exp(-\lambda x)$ (on the domain $[0, \infty)$). Then the moment-generating function is

$$E_X[\exp(tX)] = \int_0^\infty \exp(xt)\lambda \exp(-\lambda x)\ dx = \lambda \int_0^\infty \exp(xt - \lambda x)\ dx = \lambda \int_0^\infty \exp((t-\lambda)x)\ dx$$

$$= \lambda \frac{1}{t-\lambda} \exp((t-\lambda)x)\Big|_{x=0}^{\infty} = \frac{\lambda}{t-\lambda}\left(\left(\lim_{x\to\infty} \exp((t-\lambda)x)\right) - \exp((t-\lambda)\cdot 0)\right) = \frac{\lambda}{t-\lambda}(0-1)$$

$$= \frac{\lambda}{\lambda - t}$$

which is valid when $\lambda > t$ (such that the limit in the evaluation of the definite integral exists).